

# Less is More: Participation Incentives in D2D-enhanced Mobile Edge Computing under Infectious DDoS Attacks

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**Abstract**—Device-to-device (D2D) computation offloading has recently been proposed to enhance mobile edge computing (MEC) performance by exploiting spare computing resources in proximity user devices, thereby alleviating computation burdens from the network infrastructure and enabling truly pervasive edge computing. A key challenge in this new mobile computing paradigm is how to provide self-interested users with incentives to participate in D2D computing. Although incentive mechanism design has been intensively studied in the literature, this paper considers a much more challenging yet much under-investigated problem in which user incentives are complexly coupled with security risks, which is extremely important since D2D-enhanced MEC systems are vulnerable to distributed attacks, such as distributed denial of service (DDoS) attacks, due to its autonomous nature. In this paper, we build a novel mathematical framework incorporating game theory and classic epidemic models to investigate the interplay between user incentives and security risks in D2D-enhanced MEC systems under infectious DDoS attacks. A key result derived from this analysis is a phase change effect between persistent and non-persistent DDoS attacks, which is significantly different in nature from classic epidemic results for non-strategic users. Based on this, we determine the optimal reward in a contract-based incentive mechanism that the network operator should offer to users in order to maximize the operator's utility. The optimal solution exhibits an interesting "Less is More" effect: although giving users a higher reward promote more participation, it may harm the operator's utility. This is because too much participation fosters persistent DDoS attack and as a result, the *effective* participation level does not improve. Our novel model and analysis provide new and important insights and guidelines for optimizing the incentive mechanism for D2D-enhanced MEC systems as well as combating distributed attacks in the presence of strategic users. Finally, extensive simulations are carried out to verify the analytic conclusions.

**Index Terms**—Mobile edge computing, device-to-device communication, incentives, security, epidemics

## 1 INTRODUCTION

In the era of mobile computing and Internet of Things, a tremendous amount of data is generated from massively distributed sources and require timely processing to extract its maximum value. Further, many emerging applications,

such as mobile gaming and augmented reality, are delay sensitive and have resulted in an increasingly high computing demand that frequently exceeds what a single mobile device can deliver. Although cloud computing enables convenient access to a centralized pool of configurable computing resources, moving all the distributed data and computationally intensive applications to clouds (which are often physically located in remote mega-scale data centers) is simply out of the question, as it would not only pose an extremely heavy burden on today's already-congested backbone networks but also result in (sometimes intolerable) large transmission latencies that degrade the quality of service.

As a remedy to the above limitations, mobile edge computing (MEC) [1][2] has recently emerged to enable processing (some) workloads locally at the network edge without moving them to the cloud. Base stations (BSs) empowered with computing and storage capabilities via edge server co-location are considered as a key enabler of MEC [2], which can serve users' requests as a substitute of clouds, while significantly reducing the transmission latency as they are placed in the proximity of end users. Nonetheless, compared to mega-scale data centers, edge servers co-located with BSs are still limited in their computing resources. As a result, they are unable to fulfill all computation requests when they are overloaded with an excessive amount of requests from mobile users at the same time while still meeting their computation performance (e.g. latency) requirements. Moreover, since wireless environment is complicated and volatile, mobile users may experience poor wireless connection to the BS if the wireless channel is in deep fading or the user is far away from the BS, thereby preventing these mobile users from accessing the edge server resources.

In order to overcome the aforementioned drawbacks, device-to-device (D2D) communication [3] can be exploited to enhance MEC performance by providing an additional type of edge computing resources besides edge servers co-located with BSs, while avoiding global network bottlenecks. When offloading computation tasks to edge servers fails in overloaded or low connectivity scenarios, mobile users can employ local computing resources on peer mobile user devices in the vicinity for computing a shared task

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mobile computing by using resource providers other than the mobile device itself to host the execution of mobile applications [10]. In the most common case, mobile cloud computing means to run an application on a resource rich cloud server located in remote mega-scale data centers, while the mobile device acts like a thin client connecting over to the remote server through 4G/Internet [11][12][10]. Recently, the edge computing paradigm [1] (a.k.a. fog computing [13], cloudlet [14], micro datacenter [15][16]) brings computing resources closer to the end users to enable ultra-low latency and precise location-awareness, thereby supporting a variety of emerging mobile applications such as mobile gaming, augmented reality and autonomous vehicles. Edge devices, such as base stations empowered with computing capability, thus become the mobile cloud. The third and much less studied approach is to consider other mobile devices themselves as resource providers [5][6][7], which is the main focus of this paper. For instance, “Hyrax” proposed in [5] explores the possibility of using a cluster of mobile phones as resource providers and shows the feasibility of such a mobile cloud. “Serendipity” [6] is another recent and prominent work that proposes and implements a framework that uses nearby devices for distributed task computation. Computation offloading is concerned with what/when/how to offload a user’s workload from its device to the edge system or cloud (see [17][14] and references therein). The problem studied in this paper is not contingent on any specific offloading technology. Rather, we study how to incentivize computation service provisioning in the presence of security risks and hence, our approach can be used in conjunction with any of the existing offloading technology.

## 2.2 Incentives in D2D

D2D communications benefit from the fact that mobile users in close proximity can establish direct communication link over the licensed band while bypassing the cellular infrastructure such as the BSs. One common form of D2D communication is the network-controlled one in which the BS manages the switching between D2D and cellular links [18]. It is envisioned that D2D communications can dramatically improve the wireless network capacity while reducing energy consumption. When D2D is used to enhance the MEC performance, it can assist in offloading the computation workload from the BSs (with co-located edge servers) while extending the network coverage [4]. Both MEC and D2D are considered as key building blocks for the next generation (5G) mobile network [19][20] where big data will be generated by massively distributed sources and needs to be processed in a timely manner. There is a general consensus in the literature that mobile users would have no incentive to participate D2D communications unless they receive satisfying rewards from the network operator [21]. However, despite the large body of work on interference management and resource allocation in D2D communications [22][23], there is much fewer works to address the problem of providing incentives for users to adopt D2D communications. A contract-based incentive mechanism is developed in [18] to let users self-reveal their preferences. Market models are developed in [21] in which Stackelberg

games and auctions are used to design incentive mechanisms for open markets and sealed markets, respectively. However, these works do not consider the security risks in D2D communications or the dynamics of the whole system.

## 2.3 MEC security and classic epidemic models

One of the greatest challenges for creating an ecosystem where all stake-holders benefit from MEC is security. Security threats, challenges, and mechanisms inherent in the edge paradigm are analyzed in [2]. This paper focuses on infectious DDoS attack, which is one of the most common attacks in computer and communication networks and can be easily initiated in a D2D-enhanced MEC system due to its autonomous, distributed and mobile nature. To model the infectious DDoS attack, we utilize the widely-adopted Susceptible-Infected-Susceptible (SIS) model [24][25][26][27][28] in epidemiologic research, which is a standard stochastic model for virus infections. Existing works in this regard can be divided into two categories. The first category adopts a mean-field approximation to study networks consisting of a large number of individuals [24][26]. The second category tries to understand the influence of graph characteristics on epidemic spreading when users are interacting over a fixed topology [25][27][28]. Since mobile users are numerous and server-requester matching is random due to user mobility and task arrival processes, the mean-field approach provides a good model approximation for the D2D-enhanced MEC system. A classic result of these models is that there exists a critical effective spreading rate below which the epidemic dies out [24][25]. However, all these works study non-strategic users who are following a prescribed strategy. The present paper significantly departs from this strand of literature and studies strategic agents who choose their participation levels to maximize their own utility. Recently, a few economic and game theoretical models have been developed in biological epidemiology research [29][30][31] to understand infectious disease transmission when people can engage in public avoidance/social distancing/contact precautions. Whereas users in these works are solving a myopic best response problem, users in this paper are foresighted and take future costs into account of their decision making.

## 3 SYSTEM MODEL

### 3.1 Network Model and Incentive Mechanism

We consider a continuous time system and a cellular network with a large number of mobile user equipments (UEs) which have computation tasks to complete over time. These tasks are computationally intensive but delay sensitive and hence, offloading these tasks to a remote cloud for processing is infeasible since network latency is intolerable. To fulfill the delay requirement, mobile edge computing (MEC) enables UEs to offload part of their tasks to nearby edge servers (which are often co-located with the base stations). However, since the edge server has limited computation resource, it may not be able to satisfy all computation offloading task requests. In order to alleviate the computation burden from the edge server, D2D communication is used to enhance the MEC by offloading the task to peer UEs



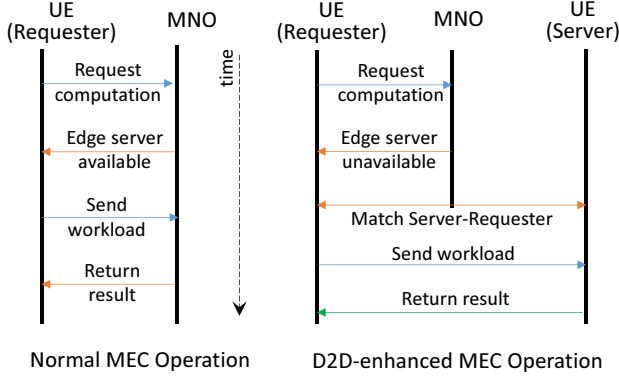


Fig. 2. Time sequence diagram of D2D-enhanced MEC operation.

in the close proximity and exploiting the spare computing resources of these UEs. To facilitate the exposition, we call the UE who requests the computing service the *requester* and the UEs who provide the computing service the *servers*. Because UEs are moving in the network, the requester-servers matching are changing over time depending on UE locations. Figure 2 shows the time sequence diagram of the D2D-enhanced MEC operation for one computation offloading task.

We assume that each task has the same workload (meaning requiring the same amount of computing resources such as CPU cycles). Tasks with different workload can be divided into smaller tasks to fulfill this requirement. Tasks are offloaded to multiple UEs in the proximity for processing. The benefit obtained by the mobile network operator (MNO) by offloading a computation task to peer UEs using D2D communication instead of to the edge server is denoted by  $b_0^t$  (e.g. reduced service congestion or edge server power consumption), which may vary over time. However, since D2D computation offloading incurs an extra cost (e.g. computation and communication power consumption) to the D2D server UE, selfish UEs are reluctant to provide the computing service. Let the cost incurred to UE  $i$  by completing one task be  $c_i$ . The costs may differ across the UEs since they may have different computing capabilities. In order to motivate participation in providing D2D computing service, the MNO offers rewards to the participating UEs, in forms of free data or monetary payments, and how much reward a UE can receive depends on its participation level.

As in [18], we consider the use of contracts as the incentive mechanism. Specifically, each UE  $i$  makes a participation-reward bundle contract  $(a^t, r(a^t))$  with the MNO at any time  $t$  where  $a_i^t$  is the D2D participation level committed by UE  $i$ , and  $r(a^t)$  is the unit time reward offered by the MNO (e.g. increased data service rate). A contract  $(a^t, r(a^t))$  requires that UE  $i$  provides computing service, when requested by the MNO, with a rate up to  $a_i^t$  tasks per unit time. The reward  $r(a^t)$  is an increasing function of  $a^t$ . Intuitively, more participation should be rewarded more and vice versa. As a practical scheme, the operator adopts a throttled linear reward function

$$r(a) = \begin{cases} r_0 a, & \forall a \leq M \\ r_0 M, & \forall a > M \end{cases} \quad (1)$$

Such a throttled scheme enables easy system implementation and is widely adopted by operators in the real world (e.g. when the reward is free data). Nevertheless, our framework and analysis can also be applied to a reward scheme without throttling. In practice, the contracts have a minimum duration. In this paper, we assume that the minimum contract duration is small enough so that UEs can effectively modify their contracts at any time in order to simplify our analysis.

### 3.2 DDoS Attack-Free Utility

Given the committed participation level  $a_i^t$ , UE  $i$  act as a D2D server at a rate of no more than  $a_i^t$  tasks per unit time from time  $t$  on until it modifies its contract. The MNO coordinates among the UEs and assigns computation tasks to UEs according to their committed participation levels. Due to randomness in the task arrival, UE mobility and the adoption of a continuous time system, we model the D2D computation request arrival to UE  $i$  as a Poisson process with rate  $a_i^t$ . The unit time utility function of UE  $i$  by choosing a participation level  $a_i^t$  is thus defined as

$$u_i(a_i^t) = v_i(r_0 a_i^t) - c_i a_i^t \quad (2)$$

where  $v_i(\cdot)$  is UE  $i$ 's evaluation function of the rewards. Clearly, selfish UEs have no incentives to participate at a level greater than  $M$  due to the throttled reward scheme and thus, we will focus on the range  $a_i^t \in [0, M]$ .

We make the following assumptions on the user evaluation function  $v(\cdot)$ .

**Assumption 1.** (1)  $v'_i(\cdot) > 0$ ,  $v''_i(\cdot) < 0$ . (2)  $v(0) = 0$ ,  $v_i(r_0 M) > c_i M$ . (3)  $v'_i(r_0 M) < c_i/r_0 < v'_i(0)$ .

The first assumption is the standard diminishing return assumption, namely the evaluation function is increasing and concave in the received reward. The second assumption states that zero participation brings zero utility, namely  $u_i(0) = 0$ , and the maximum participation yields at least non-negative utility, namely  $u_i(M) > 0$ . The third assumption is equivalent to  $\arg \max_a u_i(a) \in (0, M)$ , namely the optimal participation level lies in  $(0, M)$ . We denote  $a_i^{AF}(r_0) \triangleq \arg \max_a u_i(a)$ . In particular, the individual optimal participation level in a DDoS attack-free network thus can be computed as

$$a_i^{AF}(r_0) = \arg \max_a [v_i(r_0 a) - c_i a] = \frac{1}{r_0} (v'_i)^{-1} \left( \frac{c_i}{r_0} \right) \quad (3)$$

We further make an assumption on  $a_i^{AF}(r_0)$ .

**Assumption 2.**  $a_i^{AF}(r_0)$  is increasing in  $r_0$ .

Assumption 2 states that increasing rewards provides UEs with more incentives to participate in the D2D computation offloading as a server. This is a natural assumption and can be easily satisfied by many evaluation functions. For instance, if  $v_i(x) = \sqrt{x}$ , then  $a_i^{AF}(r_0) = \frac{r_0}{4c_i^2}$  which is increasing in  $r_0$ .

### 3.3 Infectious DDoS Attack

Participating as D2D computation server exposes UEs to DDoS attack risks since the task workload is sent directly

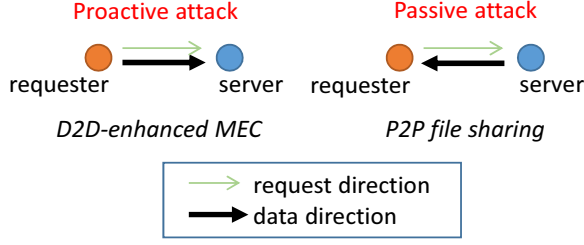


Fig. 3. Attacks are proactive in D2D computation offloading compared with P2P content sharing.

from peer UEs but not an authorized entity such as the BS and hence computer viruses can be easily planted in the exchanged data. A direct result is that any future offloading request to the compromised user will be declined, fulfilling the purpose of DDoS. What is worse is that a compromised user may become a new source of attack who may further attack other users when it becomes an offloading requester in the future.

The central idea that makes D2D computation offloading possible is distributed peer collaboration, which also forms the foundation of many other distributed peer-to-peer systems. However, unlike those systems, D2D computation offloading faces much more significant security challenges since it has a different data flow direction between the requester and the server. For instance, in peer-to-peer file sharing, file content is sent from the server to requester. As a result, attacks, which use the exchanged data as a vehicle to carry viruses, are *passive* which depend on the request arrival process. In contrast, workload data is sent from the requester to the server, resulting in much more *proactive* attacks. Figure 3 illustrates the difference between D2D computation offloading and conventional P2P systems.

In this paper, we adopt the widely-used Susceptible-Infected-Susceptible (SIS) epidemic model to model the infectious DDoS attack infection and propagation process in the D2D-enhanced MEC system. At any time  $t$ , UE  $i$  is in one of the two states  $s_i^t \in \{S, I\}$  where “S” stands for “Susceptible” (i.e. the normal state) and “I” stands for “Infected” (i.e. the compromised state). The UE state is private information and unknown to either the MNO or other UEs in the network. Otherwise, compromised UEs can be easily excluded by the MNO from D2D computing to prevent further infection and DDoS attack. If a normal UE provides D2D computing service to a compromised UE, then the normal server UE gets infected by the DDoS virus with probability  $\beta \in [0, 1]$ . We assume that an compromised server cannot infect a normal requester because the data size of the computation result is significantly smaller than the data size of the workload and hence, DDoS virus contained in the computation result returned by the server can be easily detected and removed.

Once a UE  $i$  is infected, the DDoS virus compromises the UE and the UE has to go through a recovery process. During the compromised state before recovery, UE  $i$  cannot provide any D2D computing service to other UEs and hence, the DDoS attack is completed. Moreover, UE  $i$  suffers a recovery cost  $q_i$  per unit time during this phase. However, we note that UE  $i$  can still make its own D2D computing requests

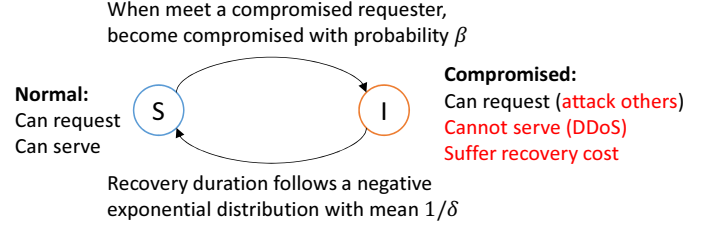


Fig. 4. UE state transition.

since the DDoS virus wants to attack other UEs through D2D communications. The recovery time duration follows a negative exponential distribution with mean  $1/\delta$ . Once the UE is recovered, it re-enters the susceptible (normal) state but may be infected/compromised again in the future. We define the *effective infection rate* as  $\tau \triangleq \beta/\delta$ .

This model follows the standard SIS model and the UE state transition is illustrated in Figure 4.

### 3.4 Problem Formulation

The objective of the MNO is to design the incentive mechanism, specifically the unit reward  $r_0$ , in order to maximize its own utility, which is defined as

$$u_0(r_0) = \mathbb{E}_{t,i}[(b_0^t - r_0)\mathbf{1}\{s_i^t = S\}a_i^t] \quad (4)$$

where the expectation is taken over the UE attributes (i.e. the distribution of the evaluation function  $v_i(\cdot)$  and D2D computing cost  $c_i$ ) and time. We assume that the benefit distribution is independent of UEs' participation decisions and identical over time and hence, the MNO's utility can be alternatively written as  $u_0(r_0) = (b_0 - r_0)\mathbb{E}_{t,i}[\mathbf{1}\{s_i^t = S\}a_i^t]$  where  $b_0 \triangleq \mathbb{E}_t b_0^t$ . Clearly,  $r_0$  has to be less than the average benefit  $b_0$ ; otherwise, the MNO has no incentives to promote D2D computing.

In a DDoS attack-free system, the MNO needs to solve the following optimization problem

$$\max_{r_0} (b_0 - r_0)\mathbb{E}_i[a_i^{\text{AF}}(r_0)] \quad (5)$$

since  $\mathbb{E}_{t,i}[\mathbf{1}\{s_i^t = S\}a_i^t] = \mathbb{E}_i[a_i^{\text{AF}}(r_0)]$ , where  $a_i^{\text{AF}}(r_0)$  is UE  $i$ 's optimal participation level given the reward mechanism  $r_0$ , which can be computed using (3). After substituting the solution of  $a_i^{\text{AF}}$  in the objective function, the problem reduces to

$$\max_{r_0} \mathbb{E}_i \left( \frac{b_0}{r_0} - 1 \right) (v_i)^{-1} \left( \frac{c_i}{r_0} \right) \quad (6)$$

This problem involves a single scalar variable and hence, it can be easily solved using numerical methods.

The presence of infectious DDoS attacks changes both the MNO's objective function and UEs' participation incentives. First, since UEs may get compromised and are not able to provide D2D computing service at times, the utility that the MNO can reap from D2D computing depends on not only the UEs' participation but also the fraction of time when they are in the normal state. Therefore, the MNO's utility is  $(b_0 - r_0)\mathbb{E}_i[\mathbf{1}\{s_i^t = S\}a_i^*(r_0)]$  and we call  $\mathbb{E}_i[\mathbf{1}\{s_i^t = S\}a_i^*(r_0)]$  the *effective participation level* of D2D computing of the system. Second, UE  $i$ 's incentive to participate in D2D computing will be determined by a

utility function  $U_i(a_i)$  that incorporates the potential future infection (which will be characterized in the next section). What much complicates the problem is the fact that the DDoS attack risk depends on not only the participation action of UE  $i$  itself but also all other UEs' participation since the infection is propagated network-wide. Therefore, the utility function  $U_i(a_i)$  should be indeed a function of all UEs' actions and we denote it by  $U_i(a_i, \mathbf{a}_{-i})$  where  $\mathbf{a}_{-i}$  is the action profile of UEs except  $i$  by convention. Formally, the MNO needs to solve an optimization problem as follows

$$\max_{r_0} (b_0 - r_0) \mathbb{E}_i[\mathbf{1}\{s_i^t = S\} a_i^*(r_0)] \quad (7)$$

$$\text{s.t. } a_i^*(r_0) = \arg \max_a U_i(a_i, \mathbf{a}_{-i} | r_0), \forall i \quad (8)$$

In the next sections, we investigate the participation incentives of UEs under the infectious DDoS attack and the resulting *effective* participation level  $\mathbb{E}_i[\mathbf{1}\{s_i^t = S\} a_i^*(r_0)]$  in order to design the optimal reward mechanism.

## 4 INDIVIDUAL PARTICIPATION INCENTIVES UNDER INFECTIOUS DDoS ATTACKS

### 4.1 Foresighted Utility

If a UE is myopic and only cares about the instantaneous utility, then the UE will simply choose a participation level  $a_i^* = \arg \max_a [v_i(r_0 a) - c_i a]$  which maximizes the instantaneous utility when it is in the normal state. However, since the infectious DDoS attack may cause potential future utility degradation, the UE will instead be foresighted and care about the *foresighted utility*. In this subsection, we define and compute the foresighted utility which is crucial for UE's participation decision making. Since the infectious DDoS attack risk depends on the probability that a server UE meets an compromised requester UE, the fraction of compromised UE in the system at any time  $t$ , denoted by  $\theta^t \in [0, 1]$ , plays a significant role in evaluating the foresighted utility. Assume that UE requester-servers matching for D2D computing is uniformly random, then the probability that a server UE  $i$  meets a compromised requester UE is exactly  $\theta^t$ . We also call  $\theta^t$  the *system compromise state* at time  $t$ . Note that  $\theta^t$  is a joint result of all UE's participation decisions. We define the foresighted utility as follows.

**Definition 1. (foresighted utility)** Given the system compromise state  $\theta$ , the foresighted utility of UE  $i$  with discount rate  $\rho$  when it takes a participation action  $a_i$  is defined as

$$U(a_i, \theta) = \rho \int_{t=0}^{\infty} \left( \underbrace{\int_{\tau=0}^t e^{-\rho\tau} u_i(a_i) d\tau}_{\text{discounted sum utility before being infected}} + \underbrace{e^{-\rho t} U_I}_{\text{discounted continuation utility after being infected}} \right) \underbrace{\theta \beta a_i e^{-\theta \beta a_i t}}_{\text{exponential distribution due to Poisson arrival}} dt \quad (9)$$

where

$$U_I = \int_{t=0}^{\infty} \left( \int_{\tau=0}^t e^{-\rho\tau} (-q_i) d\tau + \rho^{-1} e^{-\rho t} U(a_i, \theta) \right) \delta e^{-\delta t} dt \quad (10)$$

We elaborate the definition of foresighted utility below:

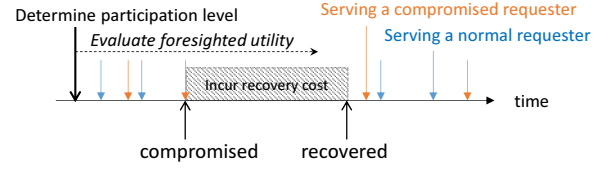


Fig. 5. Illustration of foresightedness in participation level determination.

- Suppose that UE  $i$  is compromised at time  $t_0 + t$  where  $t_0$  is the time when UE  $i$  makes the current contract, then  $\int_{\tau=0}^t e^{-\rho\tau} u_i(a_i) d\tau$  is the discounted sum utility that the UE can receive during the period  $[t_0, t_0 + t]$  before it is compromised. Notice  $e^{-\rho\tau} \leq 1$  and decreases with  $\tau$ . This means that the present value of the utility is smaller if it is realized at a later time and hence, future utility is discounted. Moreover, a larger  $\rho$  means that the discounting is greater. Using the exponential function  $e^{-\rho\tau}$  to model discounting is the standard way for continuous time systems.
- $U_I$  is the continuation utility that UE  $i$  receives once it gets compromised. During the recovery phase, UE  $i$  suffers a cost  $-q_i$  and once it is recovered, it receives again the foresighted utility  $U(a_i, \theta)$  which is discounted by  $e^{-\rho t}$  where  $t$  the duration of the recovery phase. The recovery duration  $t$  follows an exponential distribution with mean  $1/\delta$ . Here we assumed *bounded rationality* of the UE: when computing the foresighted utility, the UE believes that it will choose the *same* participation level as before once it is recovered since it expects the system compromise state to stay the same in the future. Therefore, given the same system compromise state, the optimal participation level must also be the same.
- $\theta \beta a_i e^{-\theta \beta a_i t}$  is the probability density function of being compromised at time  $t$ . The D2D request arrival is a Poisson process with rate  $a_i$  according to the committed participation level. With probability  $\theta$  UE  $i$  meets a compromised requester UE and further with probability  $\beta$  UE  $i$  is infected and compromised. As a basic property of the Poisson process, the infection process is also Poisson with arrival rate  $\theta \beta a_i$ .
- The constant  $\rho$  at the beginning on the right-hand side is a normalization term, which is due to  $\int_{t=0}^{\infty} e^{-\rho t} dt = \rho^{-1}$ .

Figure 5 illustrates the role of foresightedness plays in determining the participation level of a UE. By solving (9) and (10), the foresighted utility can be simplified to

$$U(a_i, \theta) = \frac{(\rho + \delta) u_i(a_i) - \beta \theta a_i q}{\rho + \delta + \beta \theta a_i} \quad (11)$$

The above form of foresighted utility generalizes the instantaneous utility and the time-average long-term utility and can capture a spectrum of UE behaviors by tuning the discount rate  $\rho$ . When  $\rho \rightarrow \infty$ , the UE cares about only the instantaneous utility since future utility is infinitely discounted. In this case,  $U(a_i, \theta)$  reduces to the myopic utility  $u(a_i)$ . When  $\rho \rightarrow 0$ , the UE does not discount at all and  $U(a_i, \theta)$  becomes the time-average utility  $\frac{\delta u(a_i) - \beta \theta a_i q}{\delta + \beta \theta a_i}$ ,



which is the same result by performing the stationary analysis of a continuous-time two-state Markov chain. Thus, our framework enables the investigation of D2D computing participation incentives for a wide range of application scenarios with different UE foresightedness.

## 4.2 Individual Optimal Participation Level

In this subsection, we study the optimal participation level that UE  $i$  will choose in order to maximize its foresighted utility.

**Proposition 1.** *If the system compromise state  $\theta \geq \frac{(r_0 v'_i(0) - c_i)(\rho + \delta)}{q_i \beta} \triangleq \bar{\theta}_i$ , then the optimal participation level is  $a_i^*(\theta) = 0$ . Otherwise, the optimal participation level  $a_i^*(\theta)$  is the unique solution of  $u'_i(a_i)(\rho + \delta \beta \theta a_i) - u_i(a_i)\beta\theta - \beta\theta q_i = 0$ , which increases as  $\theta$  decreases.*

*Proof.* We omit the UE index  $i$  in the subscript of  $v_i(\cdot)$ ,  $c_i$  and  $q_i$  for expositional brevity. Given the foresighted utility in (11), determining the optimal participation level boils down to investigating the first-order condition of (11). The derivative of  $U(a_i, \theta)$  is

$$U'(a_i, \theta) = (\rho + \delta) \frac{u'(a_i)(\rho + \delta + \beta\theta a_i) - u(a_i)\beta\theta - \beta\theta q}{(\rho + \delta + \beta\theta a_i)^2} \quad (12)$$

For brevity, let  $f(a_i) = u'(a_i)(\rho + \delta + \beta\theta a_i) - u(a_i)\beta\theta - \beta\theta q$ , which has the same sign of  $U'(a_i, \theta)$ . First, we have

$$\begin{aligned} f'(a_i) &= u''(a_i)(\rho + \delta + \beta\theta a_i) \\ &= r_0^2 v''(r_0 a_i)(\rho + \delta + \beta\theta a_i) < 0 \end{aligned} \quad (13)$$

Therefore,  $f(a_i)$  is monotonically decreasing. Next, we investigate the signs of  $f(M)$  and  $f(0)$ .

$$f(M) = u'(M)(\rho + \delta + \beta\theta M) - u(M)\beta\theta - \beta\theta q < 0 \quad (14)$$

The inequality is because  $u'(M) < 0$  and  $u(M) > 0$  according to Assumption 1(2). Also,

$$\begin{aligned} f(0) &= u'(0)(\rho + \delta) - u(0)\beta\theta - \beta\theta q \\ &= (r_0 v'(0) - c)(\rho + \delta) - \beta\theta q \end{aligned} \quad (15)$$

Therefore, if  $\theta < \frac{(r_0 v'(0) - c)(\rho + \delta)}{q\beta}$ , then  $f(0) > 0$ . Otherwise,  $f(0) \leq 0$ . This means that if  $\theta \geq \frac{(r_0 v'(0) - c)(\rho + \delta)}{q\beta}$ , then the optimal  $a^* = 0$  and otherwise, there exists an optimal participation level  $a^* > 0$ , which is the unique solution of

$$u'(a_i)(\rho + \delta \beta \theta a_i) - u(a_i)\beta\theta - \beta\theta q = 0 \quad (16)$$

To investigate the monotonicity of  $a_i^*$  with  $\theta$ , we rewrite the above equation as follows

$$\frac{u(a_i) + q}{u'(a_i)} - a_i = \frac{\rho + \delta}{\beta\theta} \quad (17)$$

Notice that  $u(a_i)$  does not have  $\rho, \delta, \beta$  or  $\theta$  in its expression according to (2). The first-order derivative of the left-hand side function of  $a_i$  is

$$\begin{aligned} &\frac{u'(a_i)u'(a_i) - (u(a_i) + q)u''(a_i)}{(u'(a_i))^2} - 1 \\ &= \frac{-(u(a_i) + q)u''(a_i)}{(u'(a_i))^2} > 0 \end{aligned} \quad (18)$$

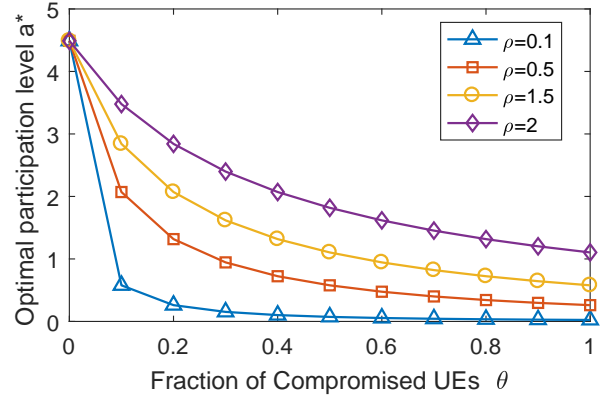


Fig. 6. Illustration of individual optimal participation level.

The last inequality is because  $u(a) > 0, \forall a \in [0, M]$  and  $u''(a) = r_0^2 v''(r_0 a_i) < 0$ . Therefore the left-hand side of (17) is monotonically increasing in  $a_i$ . Since the right-hand-side is decreasing in  $\theta$ ,  $a_i$  decreases with the increase of  $\theta$ .  $\square$

Proposition 1 can be intuitively understood. A larger system compromise state  $\theta$  implies a higher risk of getting compromised by the DDoS virus via D2D communications and hence, UE  $i$  has less incentives to participate in the D2D computing. In particular, if the system compromise state exceeds a threshold, then UE  $i$  will refrain from participating in the D2D computing. It is also evident that the operator can provide UE with more incentives to participate by increasing the reward  $r_0$ .

**Proposition 2.** *If  $\theta < \bar{\theta}_i$ , then the optimal participation level  $a_i^*(\theta)$  is increasing in  $\rho, \delta$  and decreasing in  $\beta$ .*

*Proof.* These claims are direct results of the monotonicity of the left-hand side of (17).  $\square$

Proposition 2 states that a higher DDoS attack success probability (larger  $\beta$ ) and a slower recovery speed (smaller  $\delta$ ) both decrease UE  $i$ 's incentives to participate (smaller  $\theta$ ). Importantly, Proposition 2 also reveals the impact of UE foresightedness on the participation incentives: being more foresighted (smaller  $\rho$ ) decreases the UE's participation incentives (smaller  $\theta$ ) because the UE cares more about the potential utility degradation caused by the attack. Figure 6 illustrates the individual optimal participation level for various  $\theta$  and foresightedness  $\rho$  via numerical results.

## 5 DDoS EPIDEMIC DYNAMICS

In this section, we study how the infectious DDoS attacks propagate in the system and the convergence of the system compromise state. Epidemic processes have been well investigated in the literature in different contexts. Most of this literature assumes that users are obedient and following a prescribed behavior policy. However, selfish UEs in the considered problem determine their participation levels to maximize their own foresighted utilities, thereby leading to significantly different results than conventional epidemic conclusions. To show this difference, we will first review the classical results of DDoS attack propagation in the context of the considered D2D computation offloading networks.

Then we will study the DDoS virus propagation processes for selfish UEs in two scenarios. In the first scenario, UEs observe the system compromise state at any time and hence make adaptive decisions according to the observed state. In the second scenario, UEs do not observe the system compromise state and hence have to conjecture this state based on the equilibrium analysis of a D2D computation offloading participation game.

### 5.1 DDoS Attack Propagation under a Prescribed Strategy

The evolution of the system compromise state  $\theta^t$  depends on the strategy adopted by the UEs, the attack success rate  $\beta$ , the recovery rate  $\delta$  as well as the initial state of the system  $\theta^0$ . It is obvious that if the system starts with an initial state  $\theta^0 = 0$ , then the system will remain in the state of zero compromise no matter what strategies are adopted by the UEs. Therefore, we will focus on the non-trivial case that the initial state  $\theta^0 > 0$ .

The infectious DDoS attack is said to be stationary if the system compromise state  $\theta^t$  becomes time-invariant. We denote the stationary state by  $\theta^\infty$ . The stationary compromise state reflects how the infectious DDoS attack evolves in the long-run. Existing works suggest that there exists a critical threshold  $\tau_c$  for the effective infection rate  $\tau$  such that if  $\tau < \tau_c$ , then the infectious DDoS attack extinguishes by itself even without external interventions, namely  $\theta^\infty = 0$ ; otherwise, there is a positive fraction of compromised UEs, namely  $\theta^\infty > 0$ . This result is formally presented as follows.

**Proposition 3.** *Assume that all UEs adopt a same fixed participation level  $a$ . Then there exists a critical effective infection rate  $\tau_c = \frac{1}{a}$ , such that if  $\tau \leq \tau_c$ ,  $\theta^\infty = 0$ ; otherwise,  $\theta^\infty = 1 - \frac{\delta}{\beta a}$ .*

*Proof.* Consider the compromise state dynamics given a symmetric strategy  $a$ . For any  $\theta$ , the change in  $\theta$  in a small interval  $dt$  is

$$d\theta = -\theta\delta dt + (1-\theta)\beta a dt = \theta((1-\theta)\beta a - \delta)dt \quad (19)$$

Clearly, if  $\tau > \frac{1}{a} \triangleq \tau_c$ , then for any  $\theta > 1 - \frac{\delta}{\beta a} \triangleq \theta^*$ ,  $d\theta < 0$  and for  $\theta < \theta^*$ ,  $d\theta > 0$ . Therefore, the dynamic system must converge to  $\theta^*$ . If  $\tau \leq \tau_c$ , then for any  $\theta > 0$ ,  $d\theta < 0$ . This means that the dynamic system converges to 0.  $\square$

Notice that if  $\beta < \delta$ , then for all participation levels  $a$ , we must have  $\tau \leq \tau_c$  and hence DDoS attack always extinguishes in the long-run.

### 5.2 DDoS Attack Propagation with Strategic UEs and Observable System Compromise State

In the considered D2D computing system, UEs strategically determine their participation levels and hence, the DDoS virus propagation dynamics may be significantly different. In this subsection, we investigate the infectious DDoS attack dynamics assuming that UEs can observe the system compromise state  $\theta^t$  at any time. In this case, UEs will adaptively change their participation levels by revising their contracts with the MNO according to the observed system compromise state.

For the analysis simplicity, we assume that all UEs have the same evaluation function  $v(\cdot)$ , service provision cost  $c$

and recovery cost  $q$ . We will investigate the heterogeneous case in simulations. The system dynamics thus can be characterized by the following equation,

$$d\theta^t = -\theta^t\delta dt + (1-\theta^t)\beta\theta^t a^*(\theta^t)dt \quad (20)$$

where  $a^*(\theta^t)$  is the best-response participation level given the current system compromise state  $\theta^t$  according to our analysis in the previous section. In the above system dynamics equation, the first term  $\theta^t\delta dt$  is the population mass of compromised UEs that are recovered in a small time interval  $dt$ . The second term  $(1-\theta^t)\beta\theta^t a^*(\theta^t)dt$  is the population mass of normal UEs that are compromised in a small time interval  $dt$ . The following proposition characterizes the convergence of the system dynamics.

**Proposition 4.** *There exists a critical effective infection rate  $\tau_c = \frac{1}{a^{AF}}$  such that if  $\tau < \tau_c$ , then the system compromise state converges to  $\theta^\infty = 0$ ; otherwise,  $\theta^\infty = \theta^\dagger$  where  $\theta^\dagger > 0$  is the unique solution of  $(1-\theta^\dagger)a^*(\theta^\dagger) = 1/\tau$ .*

*Proof.* The system dynamics can be rewritten as

$$d\theta^t = \theta^t[(1-\theta^t)\beta a^*(\theta^t) - \delta]dt \quad (21)$$

We are interested in the sign of  $d\theta^t$  for different values of  $\theta^t$ . Since  $\theta^t \geq 0$ , what matters is the sign of  $f(\theta^t) \triangleq (1-\theta^t)\beta a^*(\theta^t) - \delta$ . For  $\theta^t \geq \bar{\theta}$ ,  $a^*(\theta^t) = 0$  and hence,  $f(\theta^t) = -\delta < 0$ . For  $\theta^t < \bar{\theta}$ ,  $f(\theta^t)$  is decreasing in  $\theta^t$  because  $a^*(\theta^t)$  is decreasing in  $\theta^t$  according to Proposition 1. Now consider the sign of  $f(0) = \beta a^*(0) - \delta$ . According to (16),  $a^*(0)$  is the solution to  $u'(a) = 0$ , which is the same as the optimal participation level in the attack-free case. Specifically,  $a^*(0) = a^{AF}$ . If  $\beta a^{AF} - \delta < 0$ , then  $f(\theta^t) < 0$  for all  $\theta^t$ . Therefore, the system converges to  $\theta^\infty = 0$ . If  $\beta a^{AF} - \delta \geq 0$ , then there exists a unique point  $\theta^\dagger \in [0, \bar{\theta})$  such that for  $\theta^t > \theta^\dagger$ ,  $f(\theta^t) < 0$  and for  $\theta^t < \theta^\dagger$ ,  $f(\theta^t) > 0$ . This means that the system compromise state will converge to  $\theta^\dagger$ . Moreover,  $\theta^\dagger$  is the solution of  $(1-\theta)\beta a^*(\theta) - \delta = 0$ .  $\square$

Proposition 4 shows that when UEs are selfish and strategically deciding their participation levels, DDoS virus propagation also has a thresholding effect with regard to the effective infection rate. However, this thresholding effect is significantly different in nature from that when UEs are obeying a prescribed participation strategy. In the non-strategic case, the effective infection rate threshold is a function of the chosen participation level. In the strategic case, the threshold is a constant. In particular, the constant is exactly the critical threshold when UEs follow the individual optimal participation level  $a^{AF}$  in the attack-free network (see Figure 7 for an illustration).

Let us understand this thresholding effect further. As proved in Proposition 1, the individual optimal participation level under infectious DDoS attack risks is always lower than that in the attack-free network. Therefore, although the participation level is adapting over time depending on the system compromise state, it will never be higher than  $a^{AF}$ . As a result of Proposition 3, if the effective infection rate is less than  $1/a^{AF}$ , DDoS attack will extinguish by itself. When the effective infection rate becomes sufficiently large (i.e.  $\tau > 1/a^{AF}$ ), individual UEs always have incentives to choose a participation level greater than the threshold participation level that eliminates infection. This is



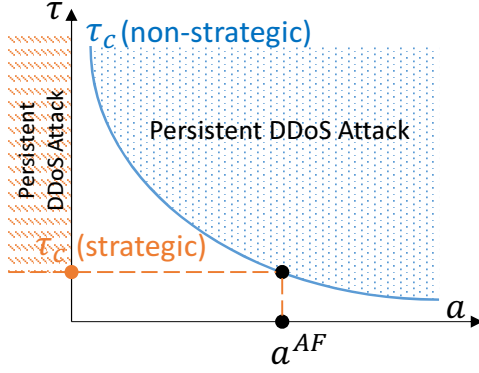


Fig. 7. Critical effective infection rates in strategic and non-strategic cases.

because unilaterally increasing the participation level does not change the network-wide epidemic dynamics since each individual UE is infinitesimal yet increases the benefit that the individual UE can obtain. Therefore infectious attacks persist.

Proposition 5 gives bounds on the convergence speed.

**Proposition 5.** *If  $\tau > \frac{1}{a^{AF}}$ , then starting from any initial system compromise state  $\theta^0 > \theta^\dagger$  (or  $\theta^0 < \theta^\dagger$ ),  $\forall \epsilon > 0$ , let  $T(\theta^0, \epsilon)$  be the time at which  $\theta^t$  decreases to  $\theta^\dagger + \epsilon \triangleq \theta_\epsilon$  (or increases to  $\theta^\dagger - \epsilon \triangleq \theta_\epsilon$ ), we have*

$$\frac{\ln \theta^0 - \ln \theta_\epsilon}{(1 - \theta^0)\beta a^*(\theta^0) - \delta} < T(\theta^0, \epsilon) < \frac{\ln \theta^0 - \ln \theta_\epsilon}{(1 - \theta_\epsilon)\beta a^*(\theta_\epsilon) - \delta} \quad (22)$$

*Proof.* The system dynamics can be written as

$$d \ln \theta^t = \frac{d\theta^t}{\theta^t} = [(1 - \theta^t)\beta a^*(\theta^t) - \delta] dt \quad (23)$$

Since  $\ln \theta^t$  is monotonically increasing in  $\theta \in (0, 1)$ ,  $\theta^0$  evolving to  $\theta_\epsilon$  is equivalent to  $\ln \theta^0$  evolving to  $\ln \theta_\epsilon$ . Since  $(1 - \theta^t)\beta a^*(\theta^t) - \delta$  is decreasing in  $\theta^t$ , the rate of absolute change  $|(1 - \theta^t)\beta a^*(\theta^t) - \delta|$  is larger if  $\theta^t$  is further away from  $\theta^\dagger$ . Therefore, before  $\ln \theta^t$  decreases (or increases) to  $\ln \theta_\epsilon$ , the rate of change is at least  $(1 - \theta_\epsilon)\beta a^*(\theta_\epsilon) - \delta$  and at most  $(1 - \theta^0)\beta a^*(\theta^0) - \delta$ . This proves the proposition.  $\square$

### 5.3 DDoS Attack Propagation with Strategic UEs and Unobservable System Compromise State

In practice, UEs often do not observe the system compromise state in real time. In this case, UEs have to conjecture how the other UEs will participate in the D2D computing service provisioning and the resulting system compromise state. To handle this situation, we formulate a population game to understand the D2D participation incentives under infectious DDoS attacks. To enable tractable analysis, we assume that there are  $K$  types of UEs. UEs of the same type  $k$  have the same evaluation function  $v_{(k)}(\cdot)$ , service provision cost  $c_{(k)}$  and recovery cost  $d_{(k)}$ . Denote the fraction of type  $k$  UEs by  $w_k$  and we must have  $\sum_{k=1}^K w_k = 1$ .

In the D2D computing participation game, each UE is a player and decides its participation level in the contract with the MNO. Since UEs do not observe the system compromise state, it is natural to assume the each UE simply adopts a

constant strategy, namely  $a_i^t = a_i, \forall t$ . The Nash equilibrium of the D2D participation game is defined as follows.

**Definition 2. (Nash Equilibrium).** *A participation action profile  $\mathbf{a}^{NE}$  is a Nash equilibrium if for all  $i$ ,  $a_i^{NE} = \arg \max_{a_i} U(a_i, \theta^\infty(\mathbf{a}^{NE}))$  where  $\theta^\infty(\mathbf{a}^{NE})$  is the converged system compromise state under  $\mathbf{a}^{NE}$ .*

First, we characterize the converged system compromise state assuming that all type  $k$  UEs choose a fixed participation level  $a_{(k)}$ .

**Proposition 6.** *Given the type-wise participation level vector  $(a_{(1)}, \dots, a_{(K)})$ , there exists a critical effective infection rate  $\tau_c = \frac{1}{\sum_{k=1}^K w_k a_{(k)}}$ , such that if  $\tau \leq \tau_c$ ,  $\theta^\infty = 0$ ; otherwise,  $\theta^\infty > 0$  is the unique solution of  $\sum_{k=1}^K \frac{\tau w_k a_{(k)}}{\tau \theta^\infty a_{(k)} + 1} = 1$ .*

*Proof.* Let  $\theta_{(k)}$  be the fraction of compromised UEs among all type  $k$  UEs. In the steady state, we have the following relation

$$\theta_{(k)}^\infty = \frac{\delta^{-1}}{\delta^{-1} + (\theta^\infty \beta a_{(k)})^{-1}} = \frac{\tau \theta^\infty a_{(k)}}{\tau \theta^\infty a_{(k)} + 1} \quad (24)$$

where the fraction of the compromised UEs among all UEs is  $\theta^\infty = \sum_k w_k \theta_{(k)}^\infty$ . It is clear from the above equation that if  $\theta^\infty = 0$ , then  $\theta_{(k)}^\infty = 0, \forall k$ . Hence,  $\theta^\infty = 0$  is a trivial solution in which  $a_{(k)}, \forall k$  can be any value. We now study the non-trivial solution  $\theta^\infty > 0$ . Rearranging the above equation, we have  $\theta_{(k)}^\infty = (1 - \theta_{(k)}^\infty) \theta^\infty \tau a_{(k)}$ . Summing up over  $k$  and multiplying by  $w_k$  on both sides, we have

$$\theta^\infty = \sum_{k=1}^K w_k \theta_{(k)}^\infty = \tau \theta^\infty \sum_{k=1}^K w_k (1 - \theta_{(k)}^\infty) a_{(k)} \quad (25)$$

This leads to

$$\tau \sum_{k=1}^K w_k (1 - \theta_{(k)}^\infty) a_{(k)} = 1 \quad (26)$$

If  $\tau \leq \tau_c = \frac{1}{\sum_{k=1}^K w_k a_{(k)}}$ , then clearly there is no non-trivial solution of  $\theta_{(k)}^\infty$  of the above equation. This implies that the only solution is  $\theta^\infty = \theta_{(k)}^\infty = 0, \forall k$ , which proves the first half of this proposition. Next, we show that if  $\tau > \tau_c$ , there indeed exists a unique solution  $\theta^\infty > 0$ . Substituting (24) into (26) yields

$$\sum_{k=1}^K \frac{\tau w_k a_{(k)}}{\tau \theta^\infty a_{(k)} + 1} = 1 \quad (27)$$

Clearly the left-hand side of the above equation  $\text{LHS}(\theta^\infty)$  is decreasing in  $\theta^\infty$ . Moreover  $\text{LHS}(0) = \tau \sum_{k=1}^K w_k a_{(k)} > 1$ , and  $\text{LHS}(1) = \tau \sum_{k=1}^K \frac{w_k a_{(k)}}{\tau a_{(k)} + 1} < \sum_{k=1}^K w_k = 1$ . Therefore, there is a unique solution of  $\theta^\infty \in (0, 1)$ .  $\square$

Proposition 6 is actually the extended version of Proposition 3, which further considers heterogeneous UEs. In the homogeneous UE case, the critical infection rate depends on the homogeneous participation level of UEs. In the heterogeneous UE case, the critical infection rate depends on the weighted average participation level of UEs.

With Proposition 6 in hand, we are able to characterize the Nash equilibrium of the D2D participation game.

**Theorem 1.** (1) The D2D participation game has a unique NE. (2) The NE is symmetric within each type, namely  $a_i^{NE} = a_{(k)}^{NE}$  for all UE  $i$  with type  $k$ . (3) If  $\tau \leq \frac{1}{\sum_{k=1}^K w_k a_{(k)}^{AF}}$ , then  $\theta^\infty = 0$  and  $a_{(k)}^{NE} = a_{(k)}^{AF}, \forall k$ . Otherwise,  $\theta^\infty > 0$  and  $\sum_{k=1}^K w_k a_{(k)}^{NE} > \tau^{-1}$ .

*Proof.* Consider type  $k$  UEs. Each UE chooses the individual optimal participation level determined by the following equation

$$u'_{(k)}(a_i)(\rho + \delta\beta\theta^\infty a_i) - u_{(k)}(a_i)\beta\theta^\infty - \beta\theta^\infty q_{(k)} = 0 \quad (28)$$

Given the same  $\beta, \delta, \theta^\infty$ , there is a unique optimal solution  $a_i^*$  according to Proposition 1. Therefore, if an equilibrium exists, UEs of the same type must choose the same participation level. To prove the existence of NE is to prove that the following function has a fixed point  $\theta^\infty$  based on our analysis in the proof of Proposition 6:

$$\theta^\infty = \tau\theta^\infty \sum_{k=1}^K w_k (1 - \theta_{(k)}^\infty) a_{(k)}(\theta^\infty) \quad (29)$$

Note that the difference from (25) is that  $a_{(k)}(\theta^\infty)$  is a function of  $\theta^\infty$  rather than a prescribed action.

First, we investigate if  $\theta^\infty = 0$  could be a fixed point. If  $\theta^\infty = 0$ , then  $a_{(k)}^* = a_{(k)}^{AF}, \forall k$ , which is the optimal participation level in the attack-free network. Therefore, if  $\tau \leq \frac{1}{\sum_{k=1}^K w_k a_{(k)}^{AF}}$ , then  $\theta^\infty = 0$  is a fixed point. Otherwise,  $\theta^\infty = 0$  is not a fixed point.

Next, we investigate if there is any  $\theta^\infty > 0$  that can be fixed point. This is to show, according to (29), if there is a solution to

$$\sum_{k=1}^K \frac{w_k \tau a_{(k)}(\theta^\infty)}{\tau \theta^\infty a_{(k)}(\theta^\infty) + 1} = 1 \quad (30)$$

Denote the left-hand side function by  $f(\theta^\infty)$ .

$$f'(\theta^\infty) = \sum_{k=1}^K w_k \tau \frac{a'_{(k)}(\theta^\infty) - \tau a_{(k)}^2(\theta^\infty)}{(\tau \theta^\infty a_{(k)}^*(\theta^\infty) + 1)^2} < 0 \quad (31)$$

The inequality is because  $a_{(k)}(\theta^\infty)$  decreases with  $\theta^\infty$  according to Proposition 1. Moreover,  $f(1) < \sum_{k=1}^K w_k = 1$  and  $f(0) = \tau \sum_{k=1}^K w_k a_{(k)}^{AF}$ . Therefore, if  $\tau > \frac{1}{\sum_{k=1}^K w_k a_{(k)}^{AF}}$ , then  $f(0) > 1$ . This means that there exists a unique positive solution  $\theta^\infty$ .  $\square$

**Corollary 1.** If UEs are homogeneous, i.e.  $K = 1$ , then the D2D computing participation game has a unique symmetric NE. Moreover, if  $\tau \leq \frac{1}{a^{AF}}$ , then  $\theta^\infty = 0$  and  $a^{NE} = a^{AF}$ ; otherwise,  $\theta^\infty = \theta^\dagger$  where  $\theta^\dagger$  has the same value given in Proposition 4.

Corollary 1 shows that, even if UEs do not observe  $\theta^t$  in real-time, the system will converge to the same state when  $\theta^t$  is observed as established in Proposition 4. The latter case indeed can be considered as that UEs are playing best response dynamics of the population game, which converges to the predicted NE. Moreover, the thresholding effect still exhibits. If the effective infection rate is sufficiently small, then the infectious DDoS attacks extinguish. Otherwise, the infectious DDoS attacks persist.

## 6 OPTIMAL REWARD MECHANISM DESIGN

Having understood UE's participation incentives in D2D computation offloading under infectious DDoS attack risks and the resulting epidemic dynamics, we are now ready to design the optimal reward mechanism. Recall the operator's optimization problem as follows

$$\max_{r_0} (b_0 - r_0) \mathbb{E}_i[\mathbf{1}\{s_i^t = S\} a_i^*] \quad (32)$$

Assume that there are  $K$  types of UEs, then the above optimization problem can be written as

$$\max_{r_0} (b_0 - r_0) \sum_{k=1}^K w_k (1 - \theta_{(k)}^\infty) a_{(k)}^*(\theta^\infty) \quad (33)$$

Note that  $\theta^\infty$ ,  $\theta_{(k)}^\infty$  and  $a_{(k)}^*$  all depend on the reward mechanism  $r_0$  even though the dependency is not explicitly expressed. Solving the above optimization problem is difficult because there are no closed-form solutions of  $\theta^\infty$ ,  $\theta_{(k)}^\infty$  and  $a_{(k)}^*$  in terms of  $r_0$  since they are complexly coupled as shown in the previous sections. Fortunately, our analysis shows that there is a structural property that we can exploit to solve this problem in a much easier way.

**Theorem 2.** The optimal reward mechanism design problem (33) under infectious DDoS attacks is equivalent to the following constrained reward mechanism design problem in the attack-free network

$$\max_{r_0} (b_0 - r_0) \sum_{k=1}^K w_k a_{(k)}^{AF}(r_0) \quad (34)$$

$$\text{s.t.} \quad \sum_{k=1}^K w_k a_{(k)}^{AF}(r_0) \leq \tau^{-1} \quad (35)$$

*Proof.* We divide the reward mechanism  $r_0$  into two categories  $\mathcal{R}_1$  and  $\mathcal{R}_2$ . Consider any reward mechanism  $r_0$ , if the resulting  $\sum_{k=1}^K a_{(k)}^*(\theta^\infty) \geq \tau^{-1}$ , then  $r_0 \in \mathcal{R}_1$ . Otherwise  $r_0 \in \mathcal{R}_2$ .

Now, according to Theorem 1, if  $r_0 \in \mathcal{R}_1$ , then we also have  $\sum_{k=1}^K w_k (1 - \theta_{(k)}^\infty) a_{(k)}^*(\theta^\infty) = \tau^{-1}$ , which is a constant that does not depend on the exact value of  $r_0$ . Therefore, the optimal  $r_0$  in  $\mathcal{R}_1$  must be the smallest possible  $r_0$  in order to maximize the operator's utility. The smallest  $r_0$  is the one such that  $\sum_{k=1}^K w_k a_{(k)}^*(\theta^\infty) = \tau^{-1}$  and  $\theta^\infty = 0$ . Since  $\theta^\infty = 0$ ,  $\sum_{k=1}^K w_k a_{(k)}^*(\theta^\infty) = \tau^{-1}$  is equivalent to  $\sum_{k=1}^K w_k a_{(k)}^{AF} = \tau^{-1}$ . This means that if  $r_0 \in \mathcal{R}_1$  is the optimal solution, it is also a feasible solution of the above constrained optimization problem.

If  $r_0 \in \mathcal{R}_2$ , then according to Theorem 1, we have  $\theta^\infty = 0$ . Again, since  $\theta^\infty = 0$ ,  $\sum_{k=1}^K a_{(k)}^*(\theta^\infty) < \tau^{-1}$  is equivalent to  $\sum_{k=1}^K w_k a_{(k)}^{AF} < \tau^{-1}$ . This also proves that if  $r_0 \in \mathcal{R}_2$  is the optimal solution, it is also a feasible solution of the above constrained optimization problem.  $\square$

Theorem 2 converts the reward mechanism optimization problem in the presence of infectious DDoS attack risks into an optimization problem in the attack-free network by simply adding a constraint. Because  $a_{(k)}^{AF}(r_0)$  can be easily computed, the converted optimization problem can be easily solved through numerical methods. In fact, since  $a_{(k)}^{AF}(r_0)$  is increasing in  $r_0$ , the search space of the optimal

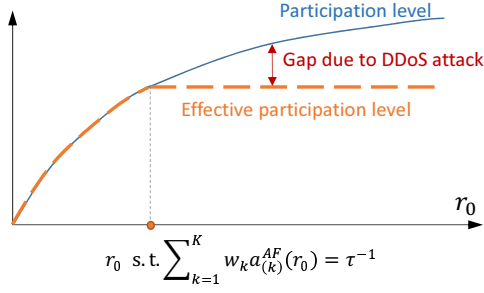


Fig. 8. Gap between participation level and effective participation level caused by DDoS attacks.

$r_0$  can be restricted to  $[0, \bar{r}_0]$  where  $\bar{r}_0$  makes the constraint binding, i.e.  $\sum_{k=1}^K w_k a_{(k)}^{AF}(\bar{r}_0) = \tau^{-1}$ .

The intuition of Theorem 2 is that the optimal reward mechanism must not promote too much participation of UEs that induce persistent DDoS attacks in the network. This is because too much participation does not improve the *effective* participation level due to UEs compromised by the attack (see an illustration in Figure 8). Since less participation requires a smaller unit reward  $r_0$ , more utility can be obtained for the MNO by employing a smaller unit reward. We call this the “less is more” effect in the D2D-enhanced mobile edge computing under infectious DDoS attack risks. Corollary 2 further compares the optimal reward mechanism with and without the infectious DDoS attack risks.

**Corollary 2.** *The optimal reward mechanism  $r^*$  for the D2D-enhanced mobile edge computing under infectious DDoS attack risks is no more than the optimal reward mechanism  $r^{AF*}$  in the attack-free network.*

*Proof.* This is a direct result of Theorem 2. If  $r^{AF*} \in \mathcal{R}_2$ , then  $r^* = r^{AF*}$ . If  $r^{AF*} \in \mathcal{R}_1$ , then  $r^* < r^{AF*}$ .  $\square$

Theorem 2 also implies that a larger effective infection rate reduces the operator’s utility.

**Corollary 3.** *The optimal utility of the MNO is non-increasing in effective infection rate  $\tau$ .*

*Proof.* This is a direct result of Theorem 2 since a larger  $\tau$  imposes a more stringent constraint in the optimization problem (34).  $\square$

Corollary 3 implies that the MNO’s utility can be improved by reducing the effective infection rate  $\tau$ . This can be done by investing in better security technologies that either reduce the attack success rate  $\beta$  or improve the recovery rate  $\delta$ . Hence, the MNO may consider jointly optimize the reward mechanism  $r_0$  and security technology that results in  $\tau$ . This is to solve the following optimization problem,

$$\max_{r_0, \tau} (b_0 - r_0) \sum_{k=1}^K w_k a_{(k)}^{AF}(r_0) - J(\tau) \quad (36)$$

$$\text{s.t.} \quad \sum_{k=1}^K w_k a_{(k)}^{AF}(r_0) \leq \tau^{-1} \quad (37)$$

where  $J(\tau)$  is the investment cost that achieves an effective infective rate  $\tau$ . Typically,  $J(\tau)$  is decreasing in  $\tau$ . It is evident that the optimal solution must

have  $\sum_{k=1}^K w_k a_{(k)}^{AF}(r_0^*) = (\tau^*)^{-1}$ . This is because if  $\sum_{k=1}^K w_k a_{(k)}^{AF}(r_0^*) < (\tau^*)^{-1}$ , then we can always choose a larger  $\tilde{\tau} > \tau^*$  such that  $J(\tilde{\tau}) < J(\tau^*)$ . This leads to a higher utility which contradicts the optimality of  $\tau^*$ . Therefore, the joint optimization problem reduces to an unconstrained problem as follows

$$\max_{r_0} (b_0 - r_0) \sum_{k=1}^K w_k a_{(k)}^{AF}(r_0) - J\left(\frac{1}{\sum_{k=1}^K w_k a_{(k)}^{AF}(r_0)}\right) \quad (38)$$

Since this problem involves a single scalar variable, it can be easily solved using numerical methods. Below we provide a simple illustrative example to show how this problem can be solved and its connection to the reward mechanism design in the attack-free network.

*Example:* Consider the homogenous agent case  $K = 1$  and let the investment cost function be  $J(\tau) = h(\frac{1}{\tau} - \frac{1}{\tau_0})$  where  $\tau_0$  is the natural effective infection rate without any security technology investment and  $h$  is a constant. In this case, the joint optimization problem becomes

$$\max_{r_0} (b_0 - h - r_0) a^{AF}(r_0) = (\hat{b}_0 - r_0)(v')^{-1} \left(\frac{c}{r_0}\right) \quad (39)$$

where  $\hat{b}_0 = b_0 - h$ . This problem indeed has the same form of the optimal reward mechanism design problem in the attack-free network (compare with (6)). The only difference is that the D2D computing benefit  $b_0$  becomes  $\hat{b}_0$  which incorporates the technology investment cost in order to reduce the effective infection rate.

## 7 SIMULATIONS

In this section, we carry out extensive simulations to verify our analytic results.

### 7.1 Simulation Setup and Time System Conversion

Although our analytic model adopts a continuous time system, we divide time into small time slots to enable the simulation. Specifically, a unit time in the continuous time system is divided into 100 slots and we simulate a large number  $T$  of slots. We simulate a number  $N = 100$  of mobile UEs moving in a square area of size  $100 \times 100$ . The number  $N$  are varied in the simulations to see how sensitive the mean-field approximation is to the finite population model. User mobility follows the random waypoint model. Specifically, when the UE is moving, it moves at a random speed between 0 and  $v_{\max}$  per slot towards a randomly selected destination. When the UE reaches the destination, it pauses for a random number of slots between 0 and  $m_{\max}$  and then selects a new destination. The parameters  $v_{\max}$  and  $m_{\max}$  control the mobility level of the network and will be varied to study the sensitivity of the random server-requester matching approximation to the actual UE mobility and matching.

In every slot, a UE has computation tasks to offload and hence becomes a requester with probability  $p$ . When a UE is not a requester, it is able to provide D2D computing service. Therefore, with probability  $1 - p$  the UE is a potential server. The number of offloading tasks of each requester in every



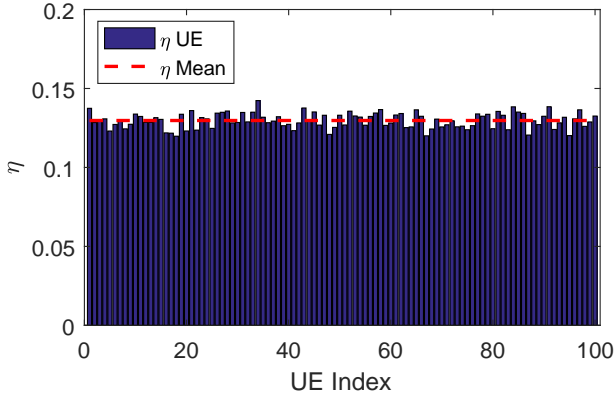


Fig. 9. Estimation of offloading request arrival.

slot is randomly selected between 1 and  $W_{max}$ . For each requester, we find the potential server UEs within a distance  $d$  of the requester where  $d$  is the D2D communication range. Suppose that UEs are obedient, then the MNO assigns one task to one of these UEs until all tasks have been assigned or the remaining tasks have to be offloaded to the closest edge server. Here, we are assuming that D2D offloading is the MNO's first choice.

The probability  $\eta$  that a potential server UE receives an offloading request in the obedient UE case can be estimated in simulations. Figure 9 illustrates the estimations for parameters  $p = 0.2$ ,  $v_{max} = 20$  and  $m_{max} = XXX$ . Estimating this probability is important for the conversion of the participation level in continuous time into its counterpart in discrete time. Specifically, a chosen participation level  $a$  per unit time in continuous time is converted to a participation probability  $\frac{a}{100(1-p)\eta}$  in each slot when the UE is a potential server in discrete time. With this conversion, in the strategic UE case, the MNO assigns one task to one of the potential server UE with probability  $\frac{a}{100(1-p)\eta}$  where  $a$  is that UE's chosen participation level until all tasks have been assigned or the remaining tasks have to be offloaded to the edge server. UEs can modify their participation decisions in every slot. Moreover, the D2D computing benefit  $b$  and cost  $c_i$  in continuous time also have to be converted to  $\frac{b}{100(1-p)\eta}$  and  $\frac{c_i}{100(1-p)\eta}$  in discrete time, respectively. The conversion is also needed for the recovery process and recovery cost. Specifically, a compromised UE recovers with probability  $\delta/100$  and the recovery cost is  $q/100$  per slot.

## 7.2 Individual UE participation

### 7.3 DDoS Epidemic Dynamics

Figure 10 illustrates the DDoS epidemic dynamics for non-strategic UEs which follow a prescribed participation strategy. Two types of UEs are considered in this simulation. Type 1 UE adopts a participation level  $a_{(1)} = 3$  and Type 2 UE adopts a participation level  $a_{(2)} = 5$ . The fractions of these two types of users are  $w_{(1)} = 0.3$  and  $w_{(2)} = 0.7$ , respectively. Therefore, the predicted critical effective infection rate  $\tau_c = 0.227$  according to Proposition 6. We fix  $\delta = 1$  and show the results for  $\beta = 0.2$  and  $\beta = 0.4$ , which correspond to  $\tau = 0.2$  and  $\tau = 0.4$ , respectively. As shown

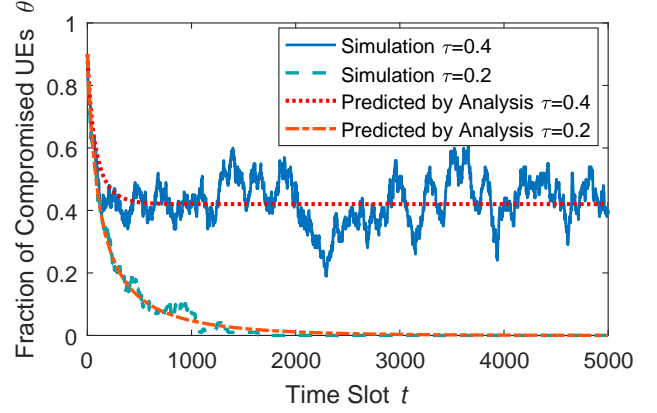


Fig. 10. Epidemic dynamics for non-strategic UEs.

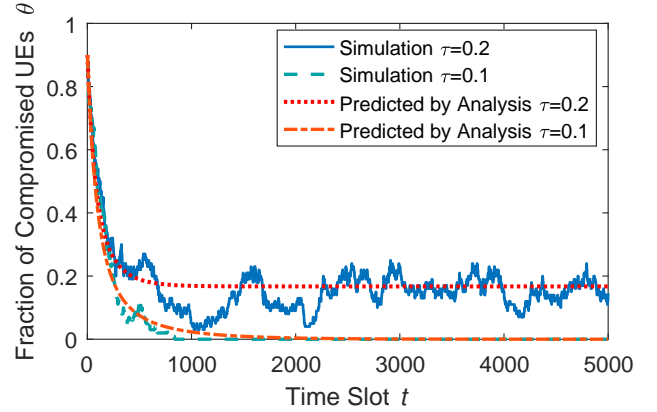


Fig. 11. Epidemic dynamics for strategic UEs.

in Figure 10, when  $\tau < \tau_c$ , DDoS attacks extinguish over time. When  $\tau > \tau_c$ , DDoS attacks persist in the system at a compromise level around 0.42. Although there are fluctuations in the simulations, the predicted dynamics by our analysis is in accordance to the simulation results.

Figure 11 illustrates the DDoS epidemic dynamics for strategic UEs for the same setting as above. The difference is that, since UEs are strategic, they will decide their participation levels by themselves. The user evaluation functions are chosen as  $v_{(1)}(x) = \sqrt{x}$  and  $v_{(2)} = 1.5\sqrt{x}$ . The costs for users are the same  $c = 0.35$ . The reward offered by the MNO is set as  $r = 2.2$ . Therefore, the critical infection rate is computed to be  $\tau_c = 0.12$  according to Theorem 1. As shown in Figure 11, DDoS attacks extinguish over time when  $\tau = 0.1$  which is smaller than  $\tau_c$  and persist when  $\tau = 0.2$  which is larger than  $\tau_c$ . Notice that in the non-strategic UE case,  $\tau = 0.2$  makes DDoS attacks extinguish, which is not true for the strategic UE case.

To better understand what is happening behind this epidemic dynamics, we show the evolution of UE participation levels (weighted average of the two types) in Figure 12. When the effective infection rate is lower, UEs tend to choose a higher participation level. Regardless of the exact value of  $\tau$ , the converged participation level does not exceed the optimal participation level  $a^{AF}$  in the attack-free network (which does not depend on  $\tau$ ). However, the converged

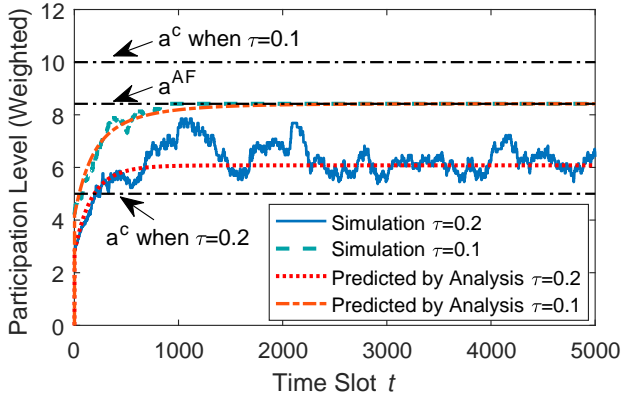


Fig. 12. Evolution of UE participation levels.

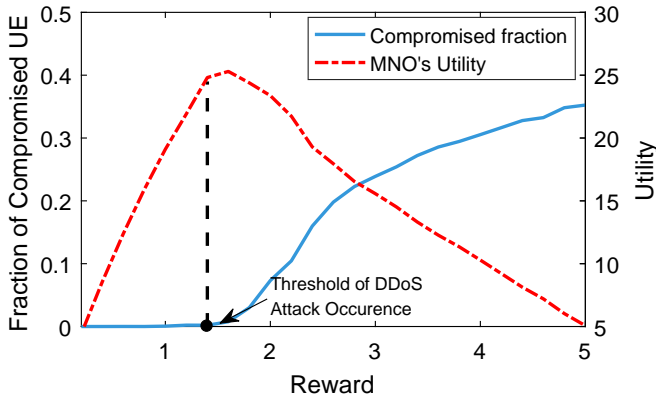


Fig. 13. Impact of reward on compromised fraction and MNO's utility.

values of the participation level are different in these two cases. When  $\tau = 0.2$ , the converged value is greater than the corresponding critical participation level  $a^c$  (which depends on  $\tau$ ) and hence, the DDoS attacks persist. When  $\tau = 0.1$ , the converged value is smaller than the corresponding critical participation level  $a^c$ , thereby eliminating the DDoS attacks.

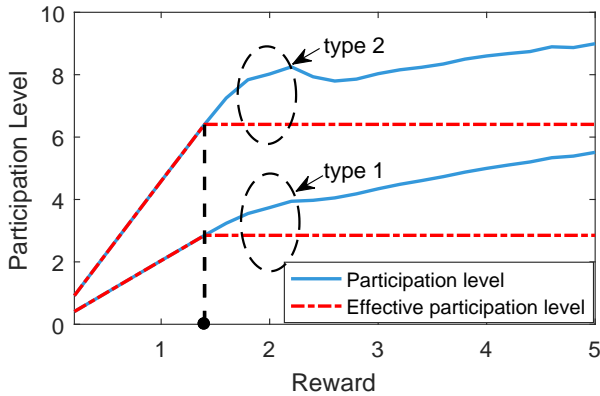
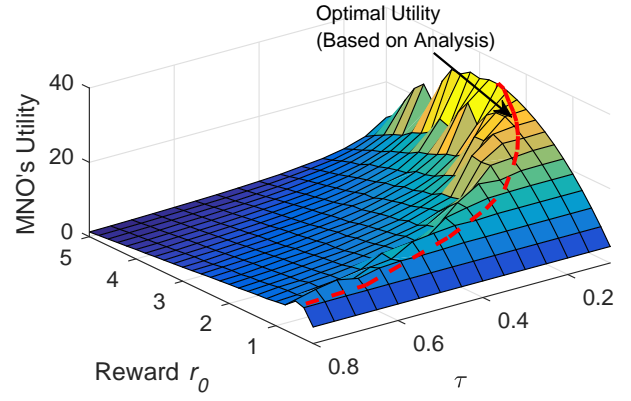
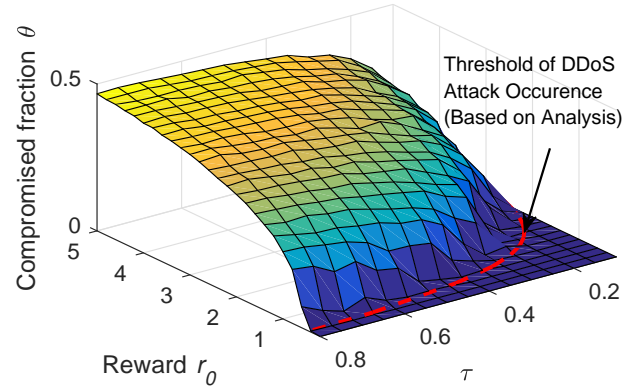


Fig. 14. Impact of reward on effective participation level.

Fig. 15. MNO's utility vs. Reward  $r_0$  and Effective Infection Rate  $\tau$ .Fig. 16. Compromised fraction  $\theta^\infty$  vs. Reward  $r_0$  and Effective Infection Rate  $\tau$ .

#### 7.4 Optimal Reward Mechanism

Now we consider the MNO's utility maximization problem. In this set of simulations, the benefit for the MNO is set as  $b = 6$ . Figure 13 shows the impact of the reward on the compromised fraction of UEs in the network as well as the MNO's utility. As the reward increases, UEs have more incentives to participate and when the reward increases to a certain point, DDoS attacks become persistent. As a result, further increasing the reward  $r_0$  decreases the MNO's utility since the effective participation level of UEs do not improve as shown in Figure 14. This is the predicted "Less is More" effect. This result is significantly important for the MNO to determine the optimal reward mechanism. Figures 15 and 16 further show the simulated MNO's utility and the compromised fraction of UEs as functions of the reward  $r_0$  and the effective infection rate  $\tau$ . The simulation results are in accordance with our analytic results.

#### 7.5 Impact of UE Mobility

Finally, we investigate the impact of UE mobility on the accuracy of our model and analysis by varying the maximum speed of UEs. Figures 17 and 18 show how the UE participation levels and the compromised fraction of UEs change with UE mobility, respectively. In our model, we assumed that a UE receives requests from other UEs

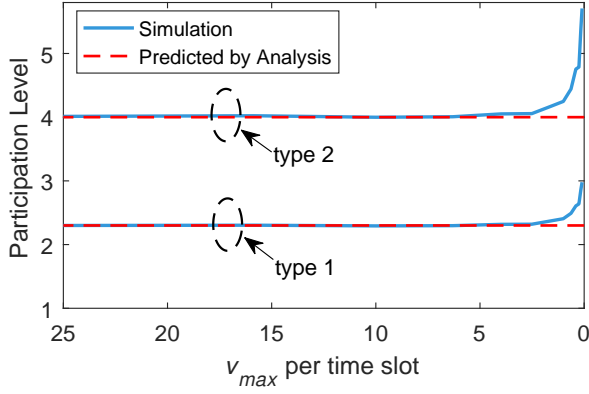


Fig. 17. Impact of mobility on the participation levels.

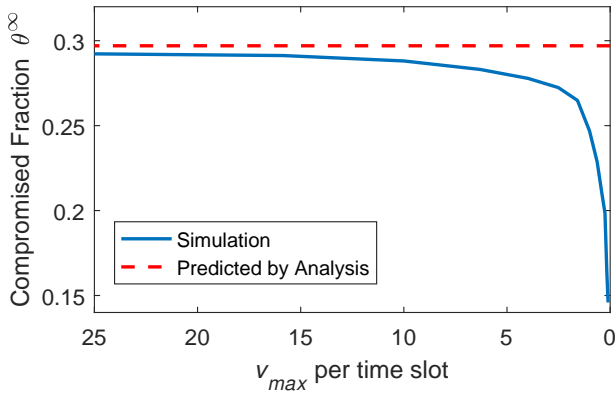


Fig. 18. Impact of mobility on the compromised fraction of UEs.

uniformly randomly, which is a good approximation when UEs' mobility is fast. However, when UEs' mobility is slow, they will more likely to have localized interactions with only a subset of UEs with high probability. For instance, in practice, people are more likely to appear at the same locations at the same time with their family, friends and colleagues. As shown in Figures 17 and 18, when UE mobility is fast, our analytic results are very much aligned with the simulation results. However, when UE mobility is slow, there is an obvious deviation of the simulation results from our analysis, suggesting that new models are needed to handle low mobility network scenarios. This is an interesting future work direction.

## 8 CONCLUSIONS

In this paper, we investigated the extremely important but much less studied incentive mechanism design problem in dynamic networks where users' incentives and security risks they face are intrinsically coupled. We adopted a dynamic non-cooperative game theoretic approach to understand how user collaboration incentives are influenced by the infectious DDoS attack risks, and how the DDoS attack risks evolve, propagate, persist and extinguish depending on users' strategic choices. This understanding allowed us to develop security-aware incentive mechanisms that are able to combat and mitigate DDoS attacks in D2D-enhanced

MEC systems. Our study leveraged the classic epidemic models, but on the other hand, it represents a significant departure from these models since users are strategically choosing their actions rather than obediently following certain prescribed rules. Our model and analysis not only provide new and important insights and guidelines for designing more efficient D2D-enhanced MEC systems but also can be adapted to solve many other challenging problems in other dynamic systems involving user incentives and security risks.

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